

Numerical Computing with Spark

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Challenges of numerical computation over big data

When applying any algorithm to big data watch for

1. Correctness
2. Performance
3. Trade-off between accuracy and performance

Three Practical Examples

- Point estimation (Variance)
- Approximate estimation (Cardinality)
- Matrix operations (PageRank)

We use these examples to demonstrate Spark internals, data flow, and challenges of implementing algorithms for Big Data.

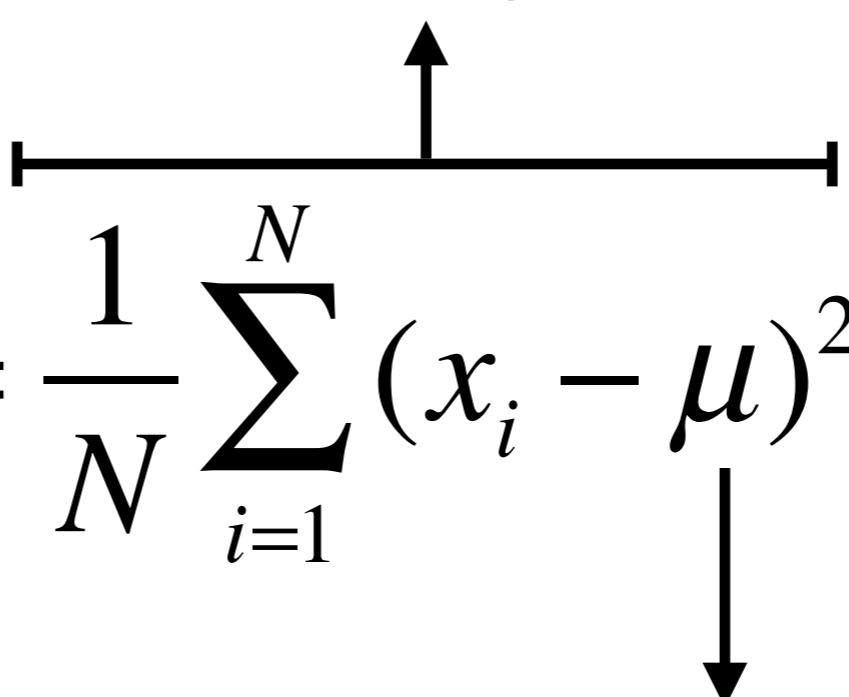
1. Big Data Variance

- The plain variance formula requires two passes over data

$$\text{Var}(X) = \frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2$$

Second pass

First pass



Fast but inaccurate solution

$$\begin{aligned} \text{Var}(X) &= E[X^2] - E[X]^2 \\ &= \frac{\sum x^2}{N} - \left(\frac{\sum x}{N} \right)^2 \end{aligned}$$

- Can be performed in a single pass, but
- Subtracts two very close and large numbers!

Accumulator Pattern

An object that incrementally tracks the variance

```
Class RunningVar {  
  var variance: Double = 0.0  
  
  // Compute initial variance for numbers  
  def this(numbers: Iterator[Double]) {  
    numbers.foreach(this.add(_))  
  }  
  
  // Update variance for a single value  
  def add(value: Double) {  
    ...  
  }  
}
```

Parallelize for performance

- Distribute adding values in map phase
- Merge partial results in reduce phase

```
Class RunningVar {  
  ...  
  // Merge another RunningVar object  
  // and update variance  
  def merge(other: RunningVar) = {  
    ...  
  }  
}
```

Computing Variance in Spark

- Use the RunningVar in Spark

```
doubleRDD  
  .mapPartitions (v => Iterator (new RunningVar (v) ) )  
  .reduce ( (a, b) => a.merge (b) )
```

- Or simply use the Spark API

```
doubleRDD.variance ()
```


2. Approximate Estimations

- Often an approximate estimate is *good enough* especially if it can be computed faster or cheaper
 1. Trade accuracy with memory
 2. Trade accuracy with running time
- We really like the cases where there is a bound on error that can be controlled

Cardinality Problem

Example: Count number of unique words in Shakespeare's work.

- Using a HashSet requires ~10GB of memory
- This can be much worse in many real world applications involving large strings, such as counting web visitors

Linear Probabilistic Counting

1. Allocate a bitmap of size m and initialize to zero.
 - A. Hash each value to a position in the bitmap
 - B. Set corresponding bit to 1
2. Count number of empty bit entries: v

$$\mathit{count} \approx -m \ln \frac{v}{m}$$

The Spark API

- Use the LogLinearCounter in Spark

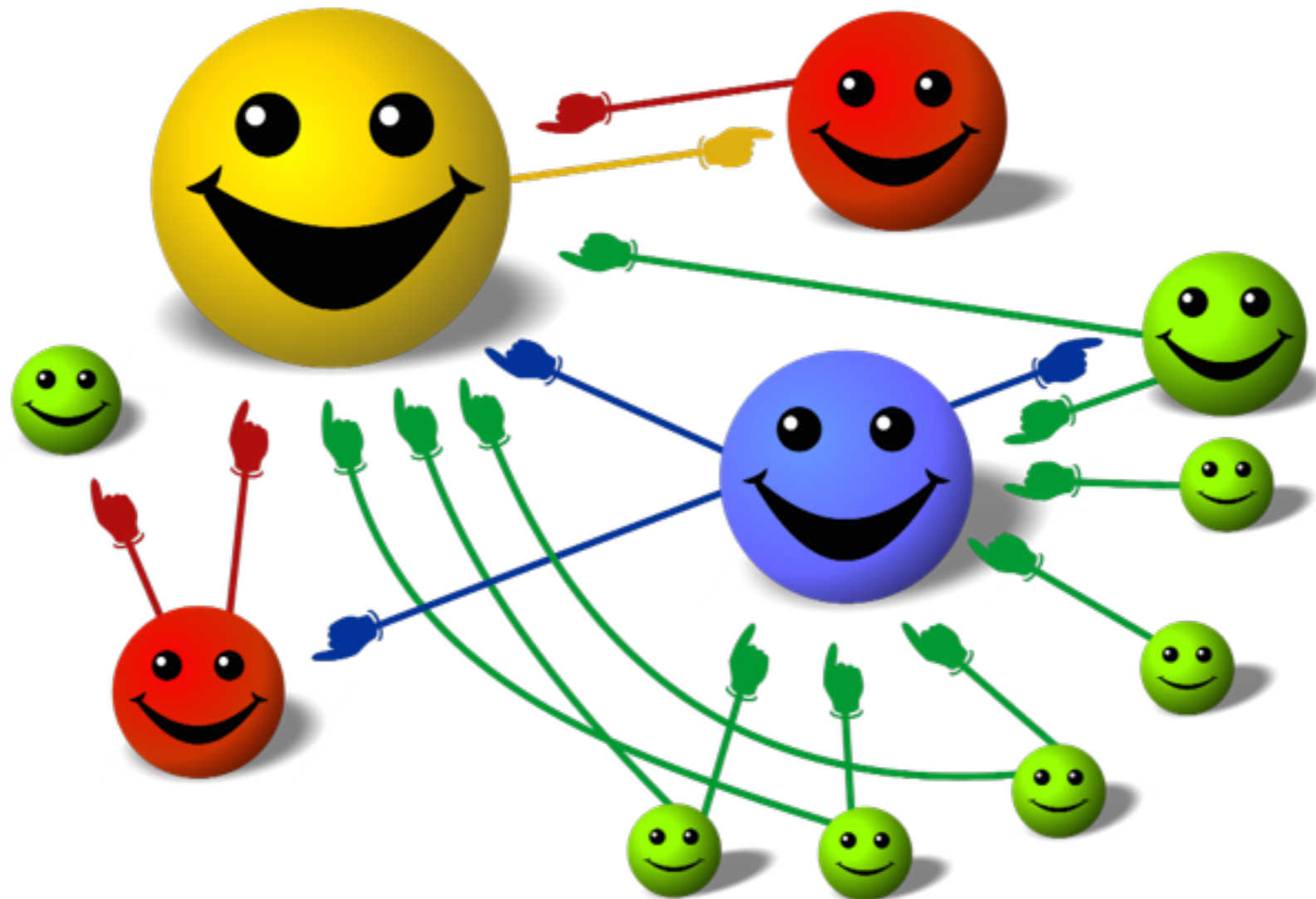
```
rdd  
  .mapPartitions (v => Iterator (new LPCounter (v) ) )  
  .reduce ( (a, b) => a.merge (b) ) .getCardinality
```

- Or simply use the Spark API

```
myRDD . countApproxDistinct (0.01)
```

3. Google PageRank

Popular algorithm originally introduced by Google



PageRank Algorithm

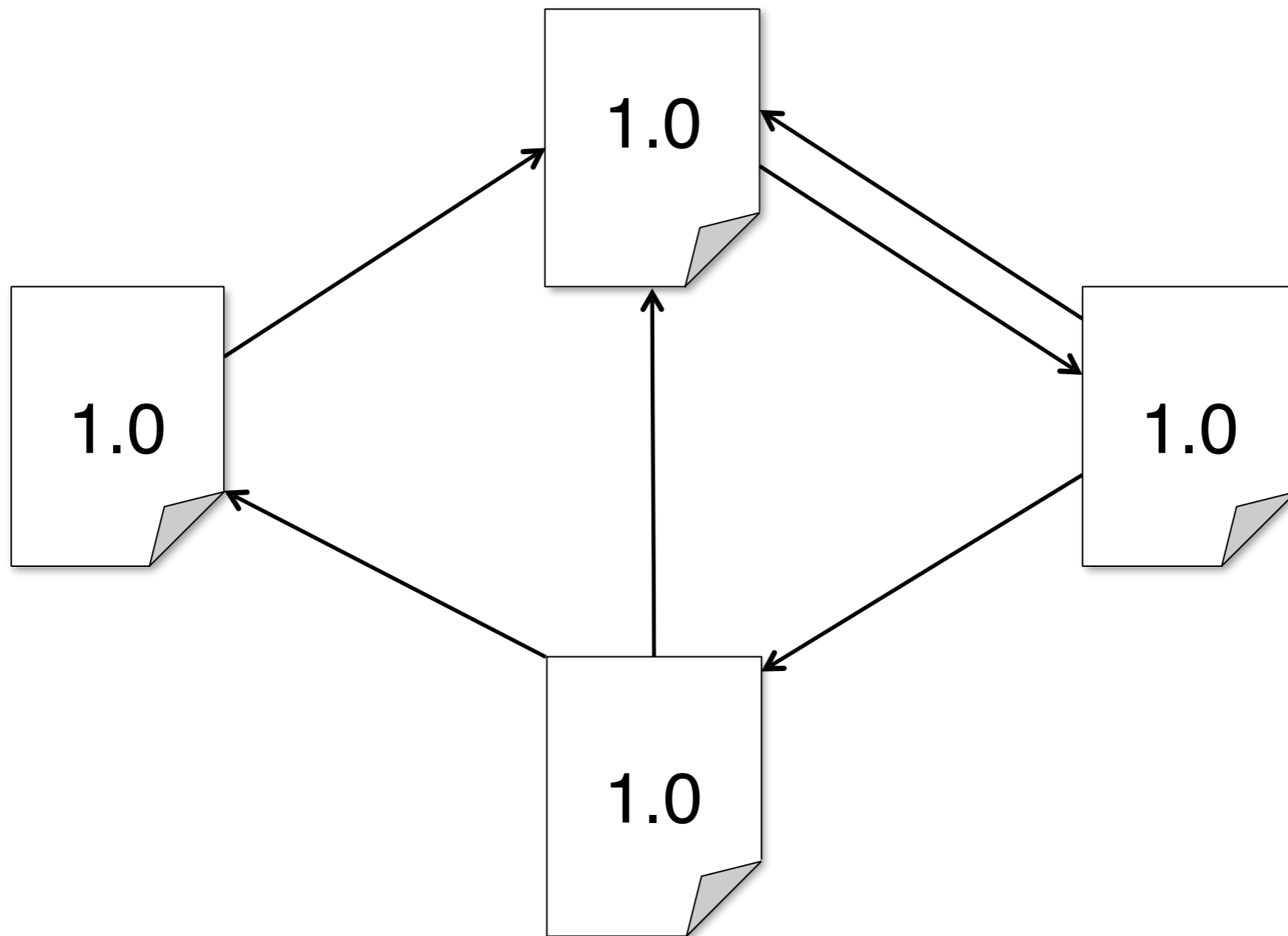
PageRank Algorithm

- Start each page with a rank of 1
- On each iteration:

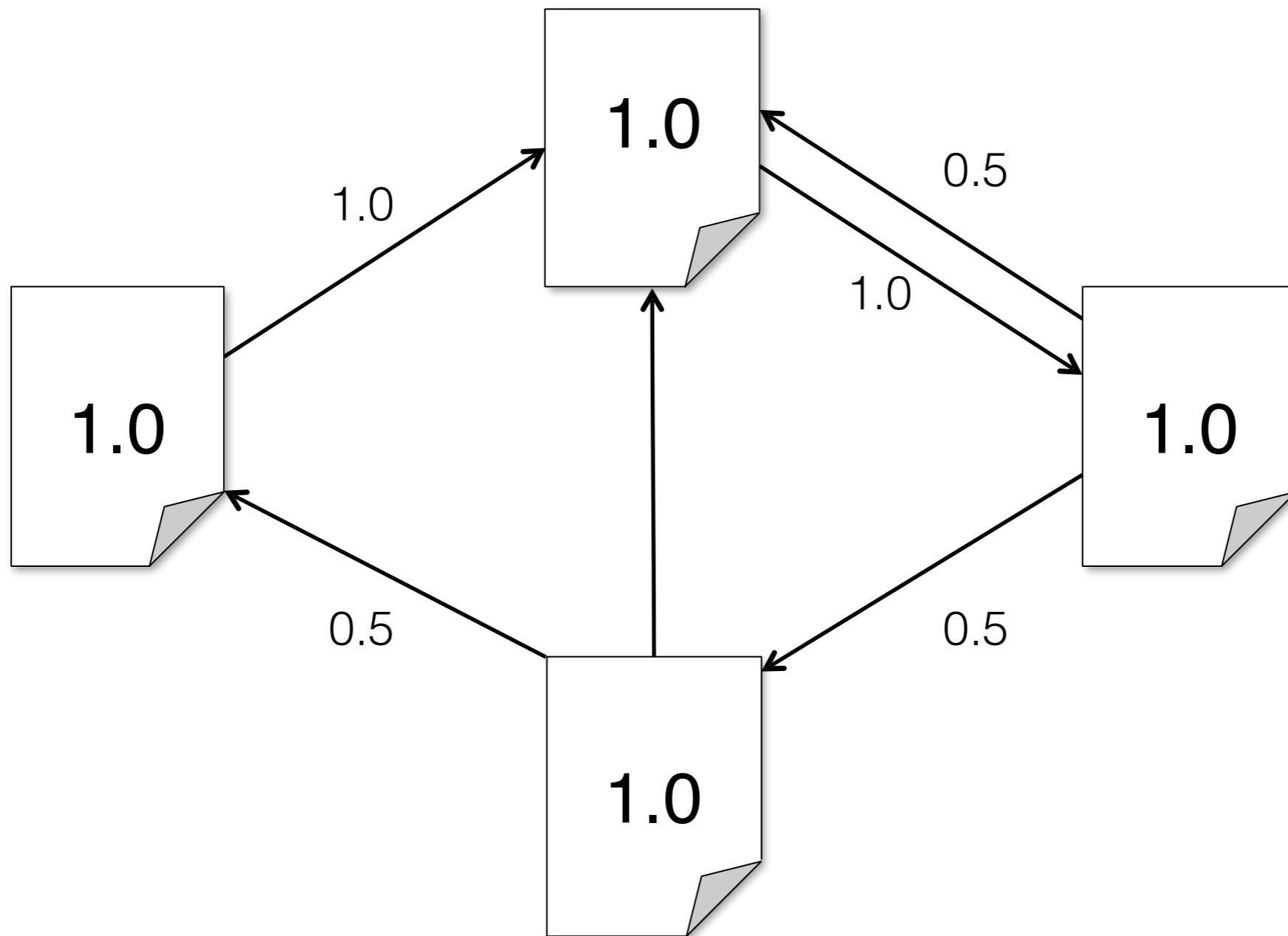
$$A. \text{ contrib} = \frac{\text{curRank}}{|\text{neighbors}|}$$

$$B. \text{ curRank} = 0.15 + 0.85 \sum_{\text{neighbors}} \text{contrib}_i$$

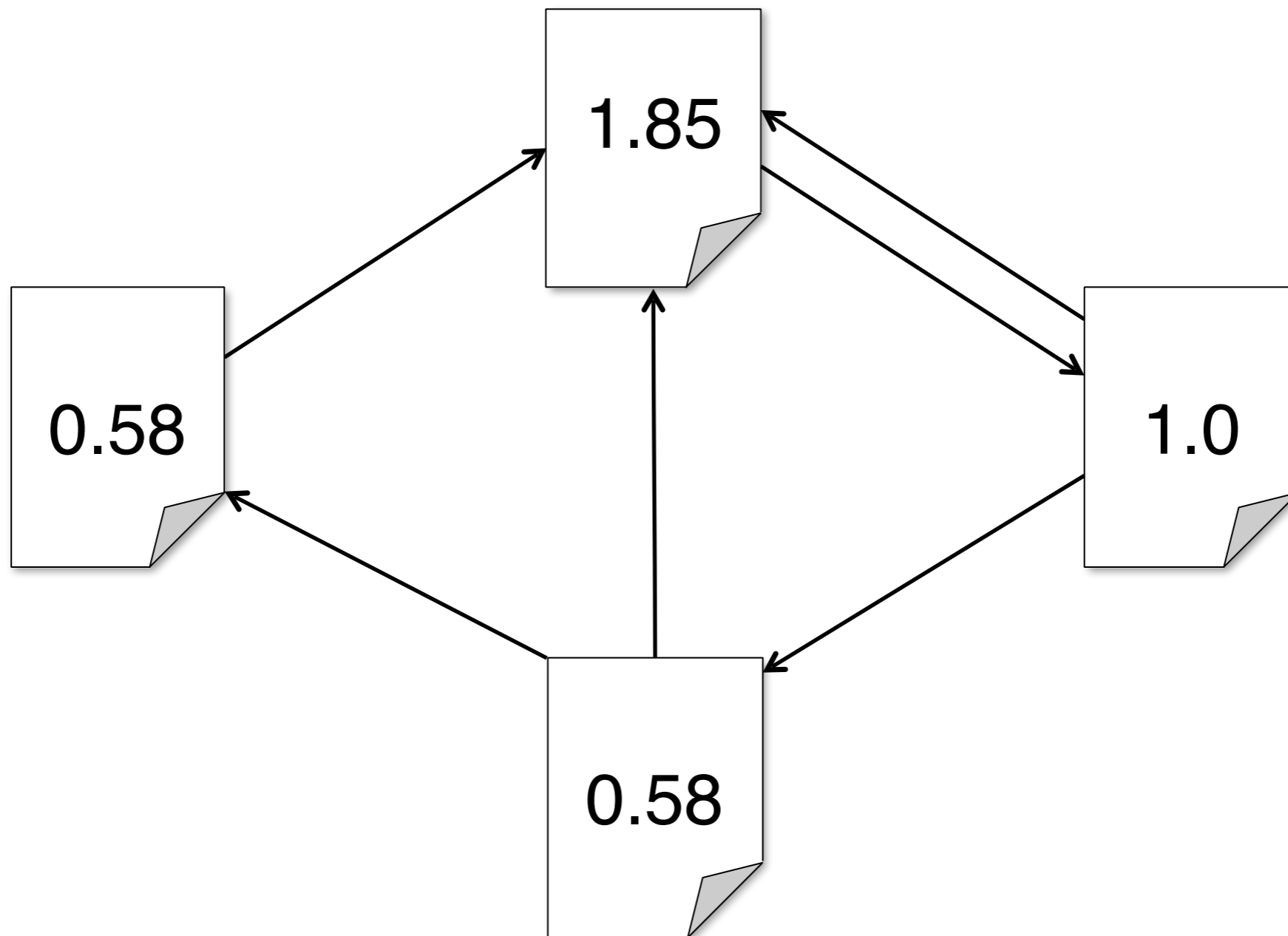
PageRank Example



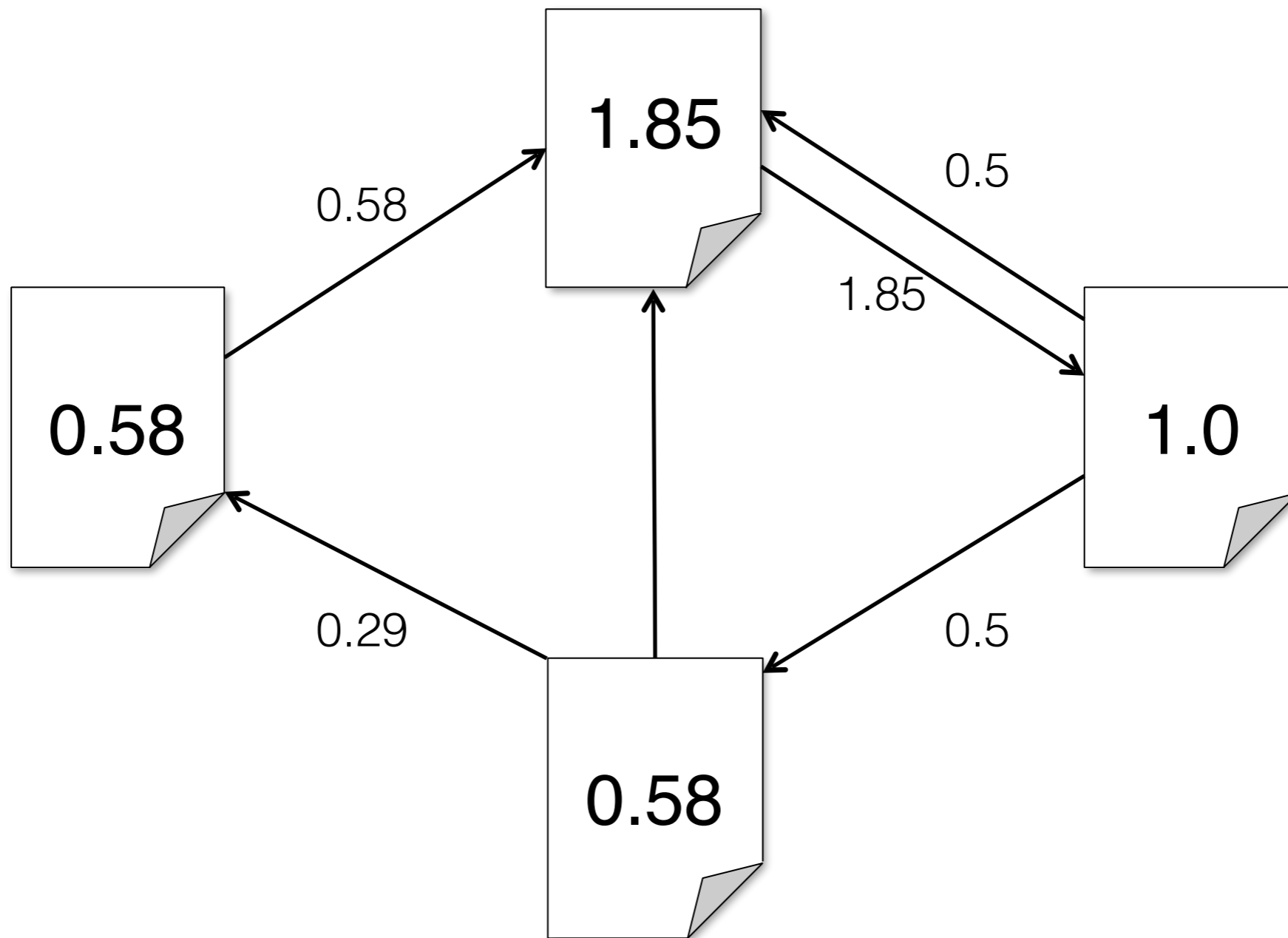
PageRank Example



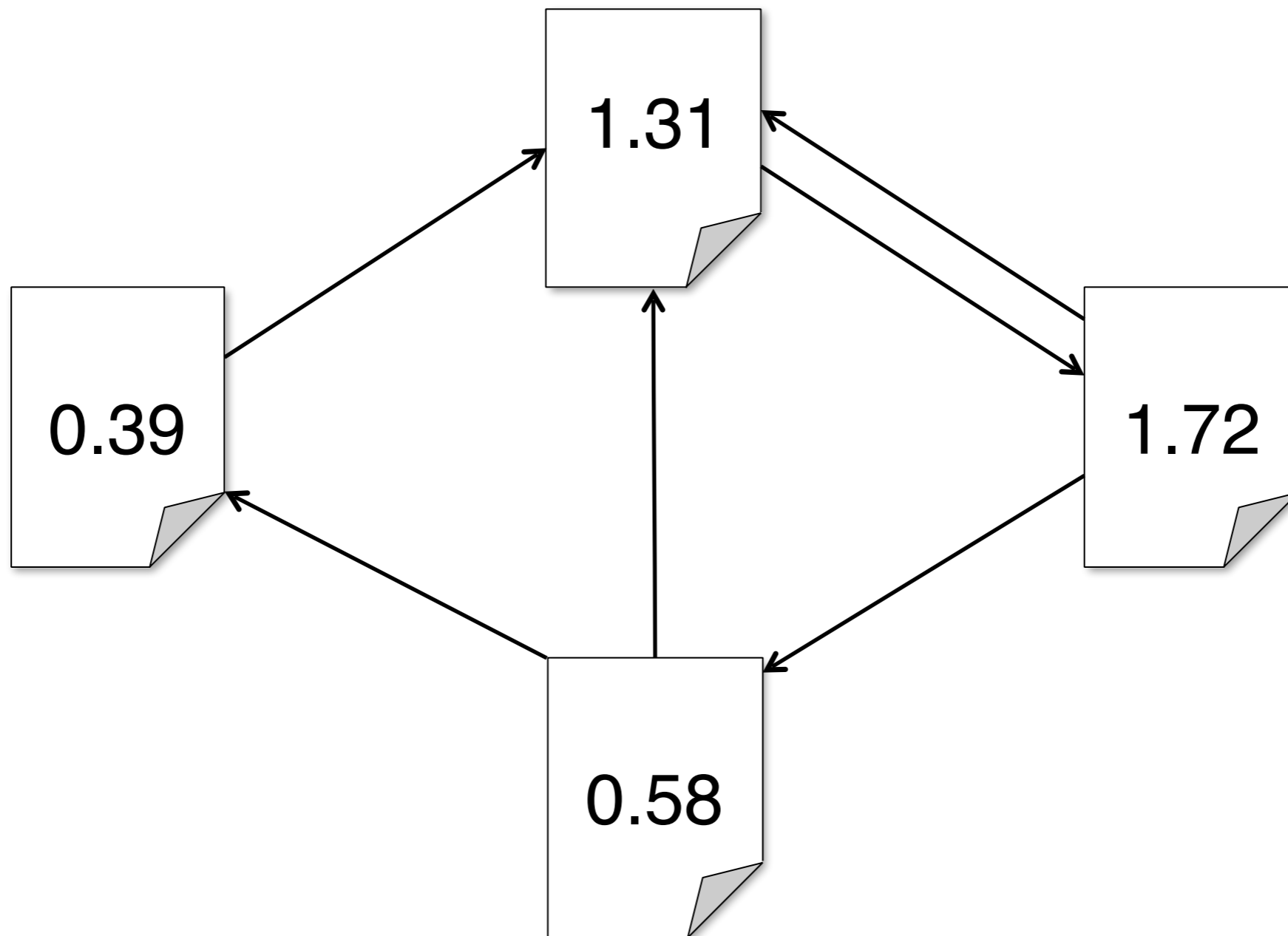
PageRank Example



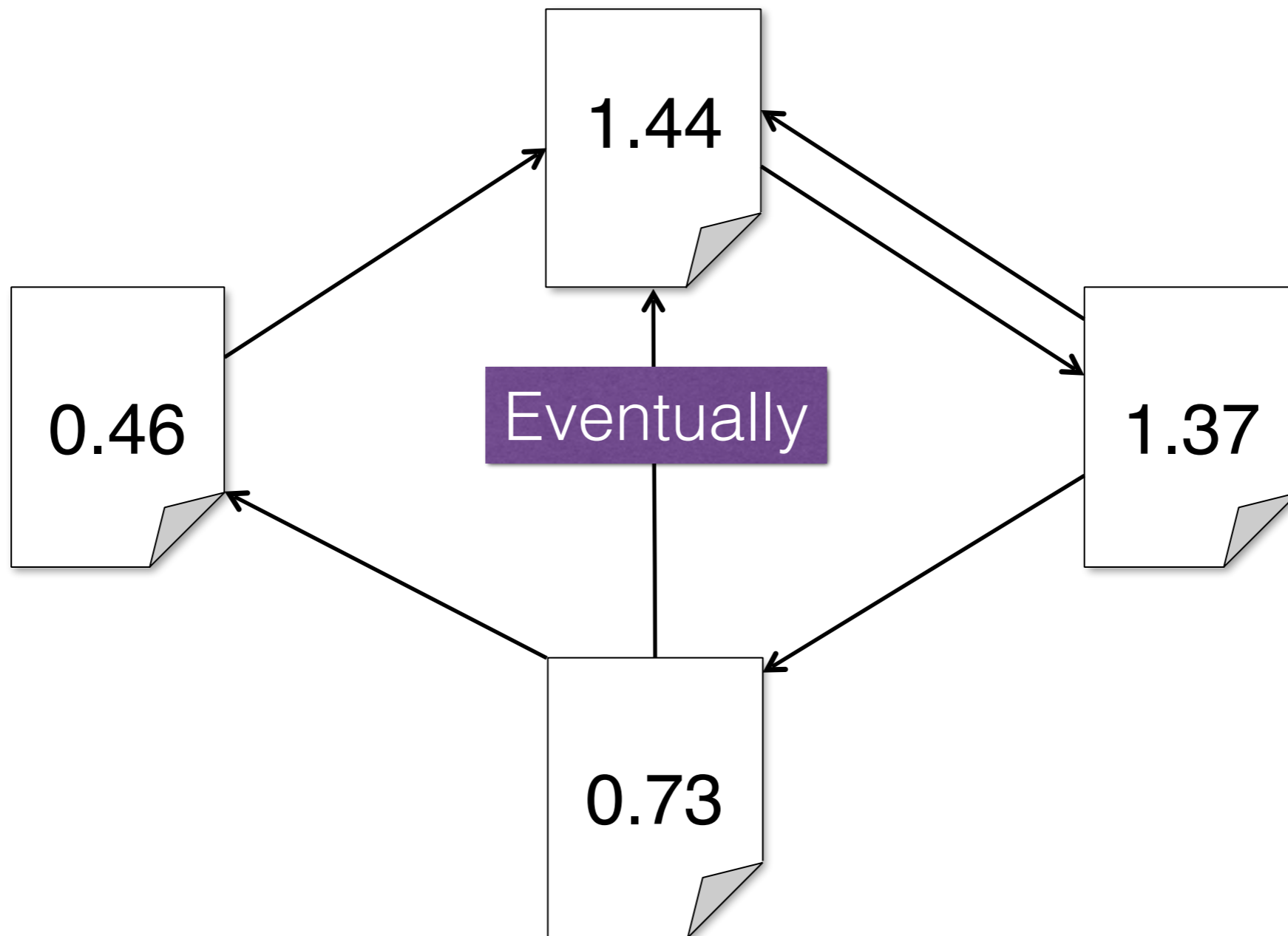
PageRank Example



PageRank Example



PageRank Example



PageRank as Matrix Multiplication

- Rank of each page is the probability of landing on that page for a random surfer on the web
- Probability of visiting all pages after k steps is

$$V_k = A^k \times V^t$$

V: the initial rank vector

A: the link structure (sparse matrix)

Data Representation in Spark

- Each page is identified by its unique URL rather than an index
- Ranks vectors (**V**): `RDD[(URL, Double)]`
- Links matrix (**A**): `RDD[(URL, List(URL))]`

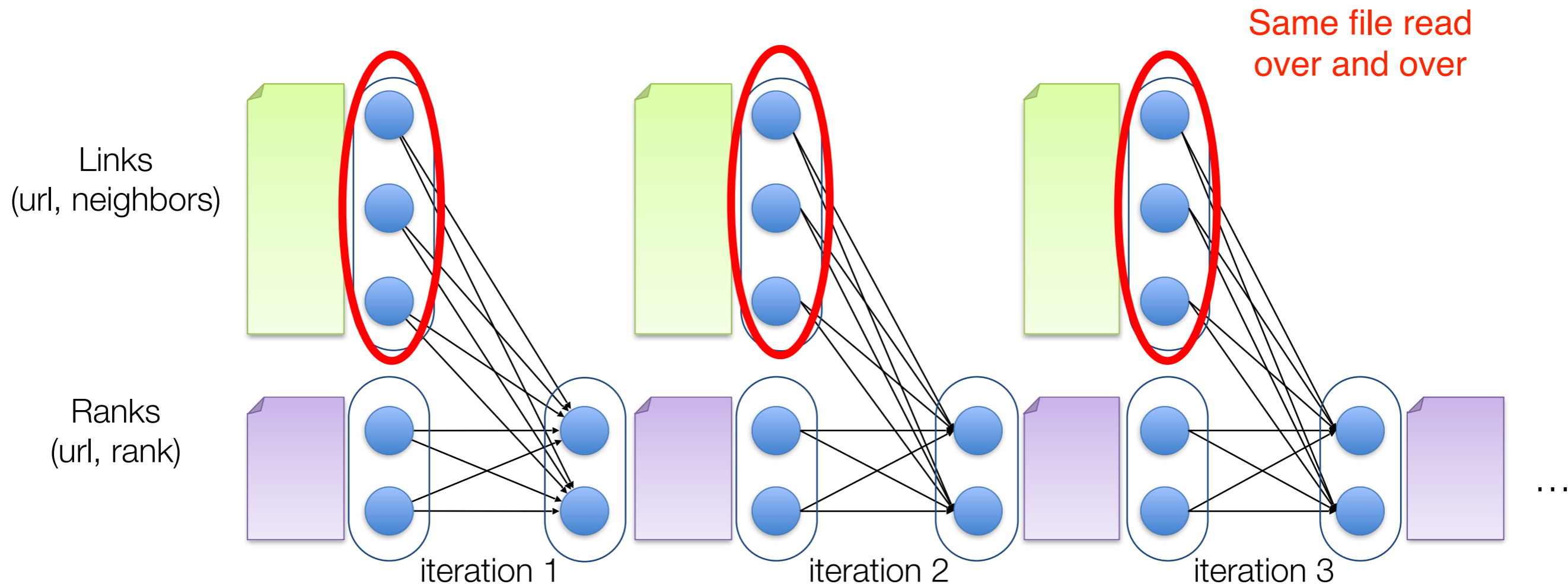
Spark Implementation

```
val links = // load RDD of (url, neighbors) pairs
var ranks = // load RDD of (url, rank) pairs

for (i <- 1 to ITERATIONS) {
  val contribs = links.join(ranks).flatMap {
    case (url, (links, rank)) =>
      links.map(dest => (dest, rank/links.size))
  }
  ranks = contribs.reduceByKey(_ + _)
    .mapValues(0.15 + 0.85 * _)
}
ranks.saveAsTextFile(...)
```

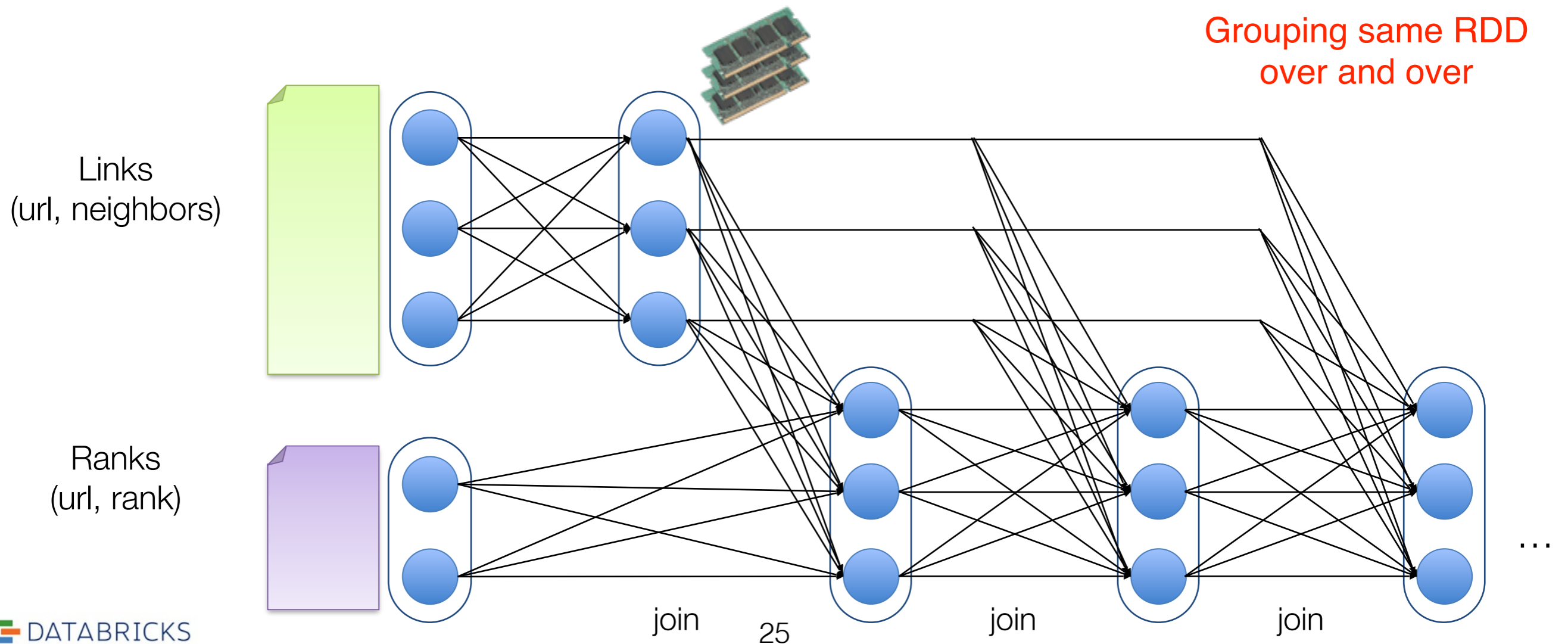
Matrix Multiplication

- Repeatedly multiply sparse matrix and vector



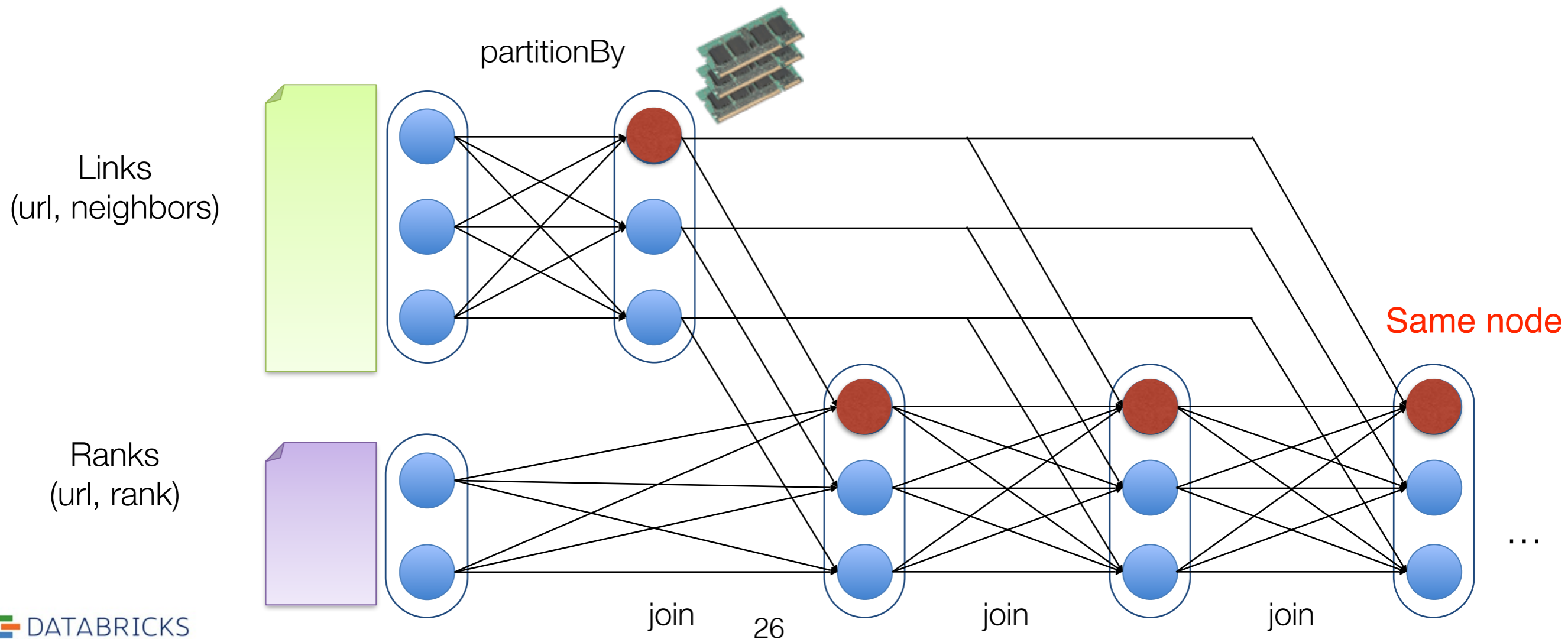
Spark can do much better

- Using `cache()`, keep neighbors in memory
- Do not write intermediate results on disk



Spark can do much better

- Do not partition neighbors every time



Spark Implementation

```
val links = // load RDD of (url, neighbors) pairs
var ranks = // load RDD of (url, rank) pairs

links.partitionBy(hashFunction).cache()

for (i <- 1 to ITERATIONS) {
  val contribs = links.join(ranks).flatMap {
    case (url, (links, rank)) =>
      links.map(dest => (dest, rank/links.size))
  }
  ranks = contribs.reduceByKey(_ + _)
    .mapValues(0.15 + 0.85 * _)
}
ranks.saveAsTextFile(...)
```

Conclusions

When applying any algorithm to big data watch for

1. Correctness
2. Performance
 - Cache RDDs to avoid I/O
 - Avoid unnecessary computation
3. Trade-off between accuracy and performance

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